Limit and Shakedown Loads for 90-Degree Mitred Pipe Bends under Combined Internal Pressure and In-Plane Bending

By

Ahmed Gaber Mohamed Afify Korba

A Thesis Submitted to the Faculty of Engineering at Cairo University in Partial Fulfillment of the Requirements for the Degree of Master of Science

In

MECHANICAL DESIGN AND PRODUCTION ENGINEERING

FACULTY OF ENGINEERING, CAIRO UNIVERSITY

GIZA, EGYPT

2012
Limit and Shakedown Loads for 90-Degree Mitred Pipe Bends under Combined Internal Pressure and In-Plane Bending

By

Ahmed Gaber Mohamed Afify Korba

A Thesis Submitted to the
Faculty of Engineering at Cairo University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science
In
MECHANICAL DESIGN AND PRODUCTION ENGINEERING

Mohammad M. Megahed
Professor of Solid Mechanics
Dept. of Mechanical Design and Production
Faculty of Engineering, Cairo University, Egypt

Mohamed M. Nassar
Professor of Engineering Mathematics
Faculty of Engineering, Cairo University, Egypt

Hany F. Abdalla
Assistant Professor of Mechanical Design and Solid Mechanics
Dept. of Mechanical Engineering
School of Sciences and Engineering
The American University in Cairo

FACULTY OF ENGINEERING, CAIRO UNIVERSITY
GIZA, EGYPT

2012
Limit and Shakedown Loads for 90-Degree Mitred Pipe Bends under Combined Internal Pressure and In-Plane Bending

By

Ahmed Gaber Mohamed Afify Korba

A Thesis Submitted to the Faculty of Engineering at Cairo University in Partial Fulfillment of the Requirements for the Degree of Master of Science in MECHANICAL DESIGN AND PRODUCTION ENGINEERING

Approved by the Examining Committee:

____________________________________
Prof. Dr. Mohamed M. Megahed, Thesis main Advisor

____________________________________
Prof. Dr. Maher Y.A. Younan, Member

____________________________________
Prof. Dr. Abdalla S. Wifi, Member

FACULTY OF ENGINEERING, CAIRO UNIVERSITY GIZA, EGYPT

2012
# Table of Contents

Table of Contents ........................................................................................................................................ iv
List of Tables ............................................................................................................................................... vii
List of Figures ........................................................................................................................................... ix
Nomenclature ............................................................................................................................................... xviii
Acknowledgment ....................................................................................................................................... xxii
Abstract ..................................................................................................................................................... xxiii

CHAPTER 1 Introduction and Literature Review ......................................................................................... 1
  1.1. Introduction ......................................................................................................................................... 1
  1.2. Literature Review ............................................................................................................................... 3
    1.2.1. Mitred Pipe Bend Literature review ......................................................................................... 3
    1.2.2. Shakedown Analysis Literature Review .................................................................................... 12
  1.3. Rationale for Investigation ................................................................................................................ 17
  1.4. Scope of Work ..................................................................................................................................... 18
  1.5. Thesis Organization ........................................................................................................................... 19

CHAPTER 2 Finite Element Modeling of Single Mitred Pipe Bend .......................................................... 21
  2.1. Introduction ......................................................................................................................................... 21
  2.2. Shell FE Modeling: ............................................................................................................................ 22
  2.3. Kitching-Bond Benchmark ................................................................................................................ 24
    2.3.1. Pressure Elastic Stresses for 90-Degree Mitred Pipe Bend ....................................................... 25
  2.4. Nilson et al’s Colapse Load Experimental Test Set-Up, FE Modeling and Results ....................... 26
CHAPTER 3  Limit Load and Ovalization For 90-Degree Mitred Pipe Bends Under Steady Internal Pressure

3.1. Introduction ........................................................................................................... 36
3.2. Single Mitred Pipe Bend ..................................................................................... 37
3.3. 2-Weld Mitred Pipe Bend .................................................................................. 44
3.4. 3-Weld Mitred pipe bend .................................................................................. 52
3.5. Smooth Pipe Bend ............................................................................................. 59
3.6. Discussion of Results ......................................................................................... 65

CHAPTER 4  Shakedown Analysis for 90-Degree Mitred Pipe Bends Using The Simplified Technique

4.1. Introduction ........................................................................................................... 72
4.2. Methodology of Shakedown Determination ....................................................... 73
4.3. Shakedown Analysis For 90-Degree Single Mitred Pipe Bend ....................... 77
4.4. Shakedown Analysis For 90-Degree 2-Weld Mitred Pipe Bend ..................... 80
4.5. Shakedown Analysis For The 90-Degree 3-Weld Mitred Pipe Bend ............... 82
4.6. Shakedown Analysis For The 90-Degree Smooth Pipe Bend ......................... 85
4.7. Discussion of Shakedown Results .................................................................... 87

CHAPTER 5  Prediction of Ratchet Boundary With Application to 90-Degree Smooth Pipe Bend ........................................................................................................................................................................... 93

5.1. Introduction ........................................................................................................... 93
5.2. Gokhfeld Concept and UMY Methodology ...................................................... 94
5.3. Comparison with Analytical Description of Ratchet Boundary ....................... 97
5.3.1. Two-Bar Problem Analysis: .......................................................................... 97
5.3.2. Bree Cylinder................................................................. 100

5.4. Ratcheting Boundary for Smooth Pipe Bend Under Internal Pressure and Cyclic Closing In-Plane Bending Moment........................................ 104

5.5. Concluding Remarks .......................................................... 109

CHAPTER 6 Conclusions and Recommendations: ........................................... 111

6.1. Conclusions:................................................................................. 111

6.1.1. Limit Load and Ovality:................................................................. 111

6.1.2. Elastic Shakedown .................................................................. 111

6.1.3. Ratchet Boundary: .................................................................. 112

6.2. Future Work and Recommendations: ........................................... 112

References......................................................................................... 114

Appendix (A)...................................................................................... 125

Appendix (B)...................................................................................... 142

Appendix (C)...................................................................................... 146

Appendix (D)...................................................................................... 147

Appendix (E)...................................................................................... 149

Appendix (F)...................................................................................... 150
List of Tables

Table 2.1 Single Mitred Pipe Bend Dimensions ................................................................. 23
Table 2.2 Specimen nominal basic dimensions. ................................................................. 27
Table 2.3 Specimen mechanical properties. ................................................................. 28
Table 2.4 Nilson et al generated FE analysis types for collapse load determination. ..... 30
Table 2.5 Comparison between Nilson et al’s FE model and the present FE model results ................................................................................................................................................. 33

Table 3.1 2-weld Model Material Properties and Dimensions ................................................. 45
Table 3.2 The 3-weld mitred pipe bend model material properties and dimensions ...... 54
Table 3.3 The smooth pipe bend model material properties and dimensions ............. 60

Table 4.1 Single Mitred Pipe Bend Model Specifications ......................................................... 77
Table 4.2 Single Yielding internal pressure and moment values for straight closed pipe. ............................................................................................................................................. 78
Table 4.3 Full Elastic Plastic analysis verification points for 90-degree single Mitred pipe bend and the observed failure mod corresponding to each point. ...................... 79
Table 4.4 2-weld Model Material Properties and Dimensions ............................................. 80
Table 4.5 Full elastic plastic analysis verification points for 90-degree 2-weld Mitred pipe bend under cyclic in-plane closing bending moment steady internal, and the observed failure mode corresponding to each point. ................................. 82
Table 4.6 The 3-weld Mitred pipe bend model material properties and dimensions ..... 83
Table 4.7 Full elastic plastic analysis verification points for 90-degree 3-weld Mitred pipe bend under cyclic in-plane closing and opening bending moment steady internal, and the observed failure mode corresponding to each point. ................................................................. 85
Table 4.8 Smooth pipe bend model material properties and dimensions .................. 85
Table 4.9  Full elastic plastic analysis verification points for 90-degree smooth pipe bend under cyclic in-plane closing and opening bending moment steady internal, and the observed failure mode corresponding to each point. ................................................................. 87
Table 4.10  Closing in-plane bending moment.......................................................................................................................... 90
Table 4.11  Opening in-plane bending moment.......................................................................................................................... 90

Table 5.1  Elastic Stresses induced by Steady Q and Cyclic Temperature History and Application of UMY Methodology to Predict Ratchet Boundary at a given Level of σth98
Table 5.2  Application of UMY to the 2-bar problem .................................................................................................. 100
Table 5.3  Detailed analysis of Bree Problem using UMY Methodology ......................... 103
Table 5.4  Detailed Implementation of UMY Methodology to a 90-degree Smooth Pipe Bend at three test cases .................................................................................................................. 107
List of Figures

Fig. 1.1 Typical 3-weld Mitred Pipe Bend shape. ................................................................. 1
Fig. 1.2 Shape of Mitred pipe bend as presented by Wood (2008) ...................................... 2
Fig. 1.3 Mitred pipe bend publications (2008). ................................................................. 3
Fig. 1.4 Loading types as stated in Wood (2008) ............................................................... 4
Fig. 1.5 Closely and Widely Spaced Mitred Pipe Bend ...................................................... 5
Fig. 1.6 Generic interaction diagram “Bree diagram” illustrating the safe and the unsafe
operating domains under combined steady and cyclic loadings (Trân et al., 2008)........... 14

Fig. 2.1 90-Degree single mitred pipe bend geometry ......................................................... 23
Fig. 2.2 FE Mesh for the Present FE Model ........................................................................... 24
Fig. 2.3 Comparison between predicted pressure elastic stresses and test data due
Kitching and Bond (1971) for 90-degree mitre at the middle welded section ............... 26
Fig. 2.4 Experimental test arrangement for collapse load in Nilson et al. (2010) ........ 27
Fig. 2.5 Geometry of Nilson et al's (2010) 90-degree Mitre Specimen ........................... 28
Fig. 2.6 Nilson et al. Force-Displacement Test Results (2010) .............................................. 29
Fig. 2.7 Nilson et al’s FE model and Adopted Idealization of Material Stress Strain
Curve, Nilson et al.(2010) ................................................................................................. 30
Fig. 2.8 Predicted Force – Displacement Curves by Nilson et al’s FE Model (2010)....... 31
Fig. 2.9 Comparison between Nilson et al results and the thesis presented model results;
(A) Nilson plastic collapse large displacement analysis for ML material behavior, (B)
Nilson et al collapse load linear small and large displacement analyses for EPP material
behavior, (C) Present thesis plastic collapse large displacement analysis for ML material
behavior, (d) Presented model collapse load linear small and large displacement analyses
for EPP material behavior .............................................................................................. 32
Fig. 2.10 Generation of Moment-Displacement Response for End Displacement and End
Rotation at the loading end point ...................................................................................... 35
Fig. 2.11 Measured and calculated moments check .......................................................... 35

Fig. 3.1 Original dimensions for the single-mitred pipe bend middle section before loading ........................................................................................................................................... 37
Fig. 3.2 Illustration of the Twice-Elastic-Slope (TES) technique Adopted for Estimation of Limit Moment from Moment-Rotation Response ........................................................................... 38
Fig. 3.3 Moment deformation large displacement analysis for single mitred pipe bend . 39
Fig. 3.4 Limit moment for 90-degree single mitred pipe bend under in-plane closing and bending moment and internal pressure ...................................................................................................... 40
Fig. 3.5 Deformed shape at mitre weld intersection showing the load-induced ovalisation at end rotation of 0.25 radians and various Levels of internal pressure ........................................... 41
Fig. 3.6 In-plane moment response versus load induced ovalisation parameters in a single mitred pipe bend at various levels of internal pressure ......................................................... 42
Fig. 3.7 Relationship between load induced ovalisation parameters and end rotation in a single mitred pipe bend due to in-plane moments at various levels of internal pressure . 43
Fig. 3.8 2-weld Mitred pipe bend Model ............................................................................ 44
Fig. 3.9 Middle area and welding area representation at the 2-weld mired pipe bend.... 45
Fig. 3.10 Moment-Rotation Response for 90-degree 2-weld mitred pipe bend under combined in-plane bending and internal pressure .............................................................................. 46
Fig. 3.11 Influence of Internal pressure on limit moment for a 90-degree 2-weld mitred pipe bend .................................................................................................................................................. 46
Fig. 3.12 Deformed Shape at the mid-section for the 2-weld mitred pipe bend showing the load-induced ovalisation at 0.25 radians and various levels of internal pressure ...... 47
Fig. 3.13 Deformed Shape at the welded-section for the 2-weld mitred pipe bend showing the load-induced ovalisation at 0.25 radians and various levels of internal pressure ...... 48
Fig. 3.14 In-plane moment response versus load induced ovalisation parameters in the a 2-weld mitred pipe bend mid section at various levels of internal pressure ................. 49
Fig. 3.15  Relationship between load induced ovalisation parameters and end rotation in the 2-weld mitred pipe bend at the mid-section due to in-plane moments at various levels of internal pressure ................................................................. 50
Fig. 3.16  In-plane moment response versus load induced ovalisation parameters in the a 2-weld mitred pipe bend welded section at various levels of internal pressure .......... 51
Fig. 3.17  Relationship between load induced ovalisation parameters and end rotation in the 2-weld mitred pipe bend at the welded section due to in-plane moments at various levels of internal pressure ............................................................................... 52
Fig. 3.18  3-weld mitred pipe bend model ................................................................................................. 53
Fig. 3.19  3-weld mitred pipe bend ovality analysis point and initial values ......................... 53
Fig. 3.20  Moment-Rotation Response for 90-degree 3-weld mitred pipe bend under combined in-plane bending and internal pressure ........................................................................... 55
Fig. 3.21  Influence of internal pressure on limit moment for a 90-degree 3-weld mitred pipe bend ........................................................................................................................................ 55
Fig. 3.22  Deformed shape at the middle welded section for the 90-degree 3-weld mitred pipe bend showing the load-induced ovalisation at end rotation of 0.25 radians and various Levels of internal pressure ................................................................................. 56
Fig. 3.23  In-plane moment response versus load induced ovalisation parameters in the a 3-weld mitred pipe bend middle section at various levels of internal pressure .......... 57
Fig. 3.24  Relationship between load induced ovalisation parameters and end rotation in the 2-weld mitred pipe bend at the welded section due to in-plane moments at various levels of internal pressure ........................................................................... 58
Fig. 3.25  Smooth pipe bend geometry ................................................................................................. 59
Fig. 3.26  Moment-Rotation Response for 90-degree smooth pipe bend under combined in-plane bending and internal pressure ........................................................................... 61
Fig. 3.27  Limit moment for 90-degree smooth pipe bend under in-plane closing and bending moment and internal pressure ........................................................................... 61
Fig. 3.28  Deformed shape at the middle section for the 90-degree smooth pipe bend showing the load-induced ovalisation at end rotation of 0.25 radians and various Levels of internal pressure ......................................................................................................................... 62

Fig. 3.29  In-plane moment response versus load induced ovalisation parameters in the a smooth pipe bend middle section at various levels of internal pressure ....................... 63

Fig. 3.30  Relationship between load induced ovalisation parameters and end rotation in the smooth pipe bend middle section due to in-plane moments at various levels of internal pressure .................................................................................................................................................................. 64

Fig. 3.31 Influence of internal pressure on the limit moments for 90-degree mitred-to-smooth pipe bends with D/t = 25 .............................................................................................................................. 66

Fig. 3.32 Comparison between moment-end rotation Responses for 90-degree mitred-to-smooth pipe bends with D/t = 25 in the presence of internal pressure ......................... 69

Fig. 3.33 Comparison between moment – ovalisation $\Delta D$ for 90-degree mitred-to-smooth pipe bends with D/t = 25 under zero pressure ............................................................................ 70

Fig. 3.34 Comparison between end rotation – ovalisation $\Delta D$ for 90-degree mitred-to-smooth pipe bends with D/t = 25 under zero pressure ............................................................................ 71

Fig. 4.1 Elastic analysis [single analysis step]: cyclic moment termed “M ref” monotonically applied (b) Elastic-Plastic analysis consists of two consecutive analysis steps: [1] Analysis step-1: steady load monotonically applied causing elastic stresses only, [2] Analysis step-2: cyclic load monotonically applied causing structural strains to exceed ($\varepsilon_0$) while the steady load applied in Analysis step-1 remains constant throughout the second analysis step.................................................................................................................. 74

Fig. 4.2 Two-bar Benchmark problem: Geometry and Interaction Diagram comparing the simplified technique results to that of the analytical results .............................................. 76

Fig. 4.3 Single Mitred Pipe Bend FE Model ................................................................................................................................. 77

Fig. 4.4 Interaction Diagram for a Single Mitred 90o Pipe Bend under combined steady pressure and cyclic In-plane bending momen........................................................................ 78
Fig. 4.5  Type of loading during the full elastic plastic analysis steps. ......................... 79
Fig. 4.6  2-weld Mitred pipe bend Model........................................................................... 80
Fig. 4.7  Interaction Diagram for a 2-weld 90-degree mitred Pipe Bend under combined steady pressure and cyclic In-plane bending momen................................................................. 81
Fig. 4.8  3-weld Mitred pipe bend Model........................................................................... 83
Fig. 4.9  Interaction Diagram for a 3-weld 90-degree mitred Pipe Bend under combined steady pressure and cyclic In-plane bending momen................................................................. 84
Fig. 4.10 Smooth pipe bend Model ..................................................................................... 85
Fig. 4.11 Interaction Diagram for 90-degree smooth pipe bend under combined steady pressure and cyclic In-plane bending momen................................................................................................. 86
Fig. 4.12  Shakedown limit moment comparison between closing and opening bending moment cases ................................................................................................................................. 89
Fig. 4.13 Elastic limit moment comparison between closing and opening bending moment cases ................................................................................................................................. 89
Fig. 4.14  SD limit and EL limit moments comparison between mitres and smooth bends at zero pressure................................................................................................................................. 91
Fig. 4.15 Elastic longitudinal stresses (in the direction perpendicular to the welded age) for two cases (a) In-plane bending moment case, (b) Internal pressure case. .................... 92

Fig. 5.1 Gokhfeld Fictitious yielding surface modification (1980) .................................... 94
Fig. 5.2 UMY Technique, Abou-Hanna and McGreevy (2011)...................................... 95
Fig. 5.3 Illustration of Strength Reduction according to UMY, Abou-Hanna and McGreevy (2011) ................................................................................................................................. 96
Fig. 5.4 Two-Bar Problem structure and the shape of the cyclic secondary loading applied on the two-bar structure................................................................................................................................. 97
Fig. 5.5 Ratcheting boundary prediction using the UMY technique. ................................. 99
Fig. 5.6 Bree cylinder model............................................................................................... 101
Fig. 5.7  Ratcheting boundary prediction using the UMY technique for the Bree cylinder problem. ................................................................. 102
Fig. 5.10  Smooth pipe bend model generation ................................................................. 104
Fig. 5.11  3D Model of the Pipe Wall Ring .................................................................... 105
Fig. 5.10  UMY Predicted Ratchet Boundary superimposed on Shakedown Boundary of a 90-degree smooth pipe bend as obtained in chapter-4 ........................................... 109

Fig. A.1  90-degree single Mitred pipe bend full elastic plastic analysis at (0.1Py) and (0.126My) showing a typical shakedown behavior. ................................................. 125
Fig. A.2  90-degree single Mitred pipe bend full elastic plastic analysis at (0.1Py) and (0.14My) in-plane closing bending moment showing a typical reversed plasticity behavior................................................................. 126
Fig. A.3  90-degree single Mitred pipe bend full elastic plastic analysis at (0.19Py) and (0.126My) in-plane closing bending moment showing a typical shakedown behavior. 126
Fig. A.4  90-degree single Mitred pipe bend full elastic plastic analysis at (0.19Py) and (0.14My) in-plane closing bending moment showing a typical reversed plasticity behavior................................................................................................................................................................. 127
Fig. A.5  90-degree single Mitred pipe bend full elastic plastic analysis at (0.1Py) and (0.126My) in-plane opening bending moment showing a typical shakedown behavior. 127
Fig. A.6  90-degree single Mitred pipe bend full elastic plastic analysis at (0.1Py) and (0.14My) in-plane opening bending moment showing a typical reversed plasticity behavior................................................................................................................................................................. 128
Fig. A.7  90-degree single Mitred pipe bend full elastic plastic analysis at (0.19Py) and (0.126My) in-plane opening bending moment showing a typical shakedown behavior. 128
Fig. A.8  90-degree single Mitred pipe bend full elastic plastic analysis at (0.19Py) and (0.14My) in-plane opening bending moment showing a typical reversed plasticity behavior................................................................................................................................................................. 129
Fig. A.9 90-degree 2-weld Mitred pipe bend full elastic plastic analysis at (0.154Py) and (0.173My) in-plane closing bending moment showing a typical shakedown behavior.

Fig. A.10 90-degree 2-weld Mitred pipe bend full elastic plastic analysis at (0.154Py) and (0.2My) in-plane closing bending moment showing a typical reversed plasticity behavior.

Fig. A.11 90-degree 2-weld Mitred pipe bend full elastic plastic analysis at (0.3465Py) and (0.173My) in-plane closing bending moment showing a typical shakedown behavior.

Fig. A.12 90-degree 2-weld Mitred pipe bend full elastic plastic analysis at (0.3465Py) and (0.173My) in-plane closing bending moment showing a typical ratcheting behavior.

Fig. A.13 90-degree 2-weld Mitred pipe bend full elastic plastic analysis at (0.154Py) and (0.173My) in-plane opening bending moment showing a typical shakedown behavior.

Fig. A.14 90-degree 2-weld Mitred pipe bend full elastic plastic analysis at (0.154Py) and (0.2My) in-plane closing bending moment showing a typical reversed plasticity behavior.

Fig. A.15 90-degree 2-weld Mitred pipe bend full elastic plastic analysis at (0.3465Py) and (0.173My) in-plane closing bending moment showing a typical shakedown behavior.

Fig. A.16 90-degree 2-weld Mitred pipe bend full elastic plastic analysis at (0.3465Py) and (0.173My) in-plane closing bending moment showing a typical ratcheting behavior.

Fig. A.17 90-degree 3-weld Mitred pipe bend full elastic plastic analysis at (0.2Py) and (0.189My) in-plane closing bending moment showing a typical shakedown behavior.

Fig. A.18 90-degree 3-weld Mitred pipe bend full elastic plastic analysis at (0.2Py) and (0.2My) in-plane closing bending moment showing a typical reversed plasticity behavior.
Fig. A.19  90-degree 3-weld Mitred pipe bend full elastic plastic analysis at (0.2Py) and (0.186My) in-plane closing bending moment showing a typical shakedown behavior. 134

Fig. A.20  90-degree 3-weld Mitred pipe bend full elastic plastic analysis at (0.2Py) and (0.2My) in-plane closing bending moment showing a typical ratcheting behavior. .... 135

Fig. A.21  90-degree 3-weld Mitred pipe bend full elastic plastic analysis at (0.2Py) and (0.189My) in-plane opening bending moment showing a typical shakedown behavior. 135

Fig. A.22  90-degree 3-weld Mitred pipe bend full elastic plastic analysis at (0.2Py) and (0.2My) in-plane opening bending moment showing a typical reversed plasticity behavior. ................................................................. 136

Fig. A.23  90-degree 3-weld Mitred pipe bend full elastic plastic analysis at (0.2Py) and (0.186My) in-plane opening bending moment showing a typical shakedown behavior. 136

Fig. A.24  90-degree 3-weld Mitred pipe bend full elastic plastic analysis at (0.2Py) and (0.2My) in-plane opening bending moment showing a typical ratcheting behavior. .... 137

Fig. A.25  90-degree smooth pipe bend full elastic plastic analysis at (0.21Py) and (0.22My) in-plane closing bending moment showing a typical shakedown behavior. .. 137

Fig. A.26  90-degree smooth pipe bend full elastic plastic analysis at (0.22Py) and (0.244My) in-plane closing bending moment showing a typical reversed plasticity behavior. .............................................................................. 138

Fig. A.27  90-degree smooth pipe bend full elastic plastic analysis at (0.56Py) and (0.16My) in-plane closing bending moment showing a typical shakedown behavior. .. 138

Fig. A.28  90-degree smooth pipe bend full elastic plastic analysis at (0.56Py) and (0.22My) in-plane closing bending moment showing a typical ratcheting behavior. .... 139

Fig. A.29  90-degree smooth pipe bend full elastic plastic analysis at (0.21Py) and (0.22My) in-plane opening bending moment showing a typical shakedown behavior. . 139

Fig. A.30  90-degree smooth pipe bend full elastic plastic analysis at (0.22Py) and (0.244My) in-plane opening bending moment showing a typical reversed plasticity behavior. .............................................................................. 140

xvi
Fig. A.31 90-degree smooth pipe bend full elastic plastic analysis at (0.56Py) and (0.16My) in-plane opening bending moment showing a typical shakedown behavior. 140
Fig. A.32 90-degree smooth pipe bend full elastic plastic analysis at (0.56Py) and (0.22My) in-plane opening bending moment showing a typical ratcheting behavior. ... 141

Fig. B.1 Comparison between moment – ovalization $\Delta D$ for 90-degree mitred-to-smooth pipe bends with $D/t = 25$ under 0.1Py ................................................................. 142
Fig. B.2 Comparison between end rotation – ovalization $\Delta D$ for 90-degree mitred-to-smooth pipe bends with $D/t = 25$ under zero pressure ............................................. 143
Fig. B.3 Comparison between moment – ovalization $\Delta D$ for 90-degree mitred-to-smooth pipe bends with $D/t = 25$ under 0.2Py ................................................................. 144
Fig. B.4 Comparison between end rotation – ovalization $\Delta D$ for 90-degree mitred-to-smooth pipe bends with $D/t = 25$ under 0.2Py ................................................................. 145

Fig. D.1 Geometrical bases in dividing Mitre joints ............................................. 147
Fig. D.2 Corresponding section lengths for different values of mitre joints and a bend radius of 145 mm ................................................................. 148

Fig. E.1 Full model to half model comparison for SMPB under IPC bending ............. 149
Fig. F.1 S4R geometrical representation .......................................................... 150
## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Cross sectional area</td>
</tr>
<tr>
<td>$A_{SY}$</td>
<td>Area under the original yielding curve</td>
</tr>
<tr>
<td>D</td>
<td>Pipe outer mean diameter</td>
</tr>
<tr>
<td>$D_m$</td>
<td>Mean tube diameter</td>
</tr>
<tr>
<td>E</td>
<td>Modulus of elasticity</td>
</tr>
<tr>
<td>F</td>
<td>Force</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Elements</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of the pipe Leg</td>
</tr>
<tr>
<td>$L'$</td>
<td>The Minimum required leg length to remove the effect of the end caps from the stress reading at the joints</td>
</tr>
<tr>
<td>$L_i$</td>
<td>The leg section length in the Mitred pipe bend model</td>
</tr>
<tr>
<td>$L_1$, $L_2$ and $L_3$</td>
<td>Lengths of pipe leg sections</td>
</tr>
<tr>
<td>M</td>
<td>Moment</td>
</tr>
<tr>
<td>$M_i$</td>
<td>The cyclic moment value in each increment</td>
</tr>
<tr>
<td>$M_{ref}$</td>
<td>Reference Moment value</td>
</tr>
<tr>
<td>P</td>
<td>Internal Pressure</td>
</tr>
<tr>
<td>$P_c$</td>
<td>New collapsing pressure value</td>
</tr>
<tr>
<td>$P_y^o$</td>
<td>Collapsing pressure value at zero moment</td>
</tr>
<tr>
<td>Q</td>
<td>Steady axial load</td>
</tr>
</tbody>
</table>
\( \bar{Q} \) Dimensionless axial load

\( R \) Bend radius of the curvature

\( R_m \) Mean radius of the pipe bend

\( S_y \) Yield strength

\( S_{r,ab} \) Deviatoric Residual Stress Tensor

\( T \) Temperature

\( d_m \) Mean diameter

\( d\alpha \) Incremental deviatoric back stress

\( d\bar{\varepsilon}^{pl}, d\varepsilon_p \) Incremental equivalent plastic strain

\( i \) Increment number

\( t \) thickness

\( \Delta T \) Temperature difference

\( \Delta \bar{T} \) Dimensionless temperature difference

\( \Psi \) Dimensionless area ratio

\( \alpha \) Pipe bend angle

\( \delta \) Kronecker delta

\( \varepsilon \) Strain

\( \varepsilon^{pl} \) Equivalent plastic strain

\( \varepsilon_o \) Initial yield strain
υ Poisson’s ratio

\( \sigma_E \) Elastic stress components

\( \sigma_{ELP_i} \) Elastic plastic stress components for each increment (i)

\( \sigma_p \) Von-Mises stresses due to internal pressure

\( \sigma_{r_{ab}} \) Residual Stress Tensor

\( \sigma_{r_x} \) Residual normal stress component in x-direction

\( \sigma_{r_y} \) Residual normal stress component in y-direction

\( \sigma_{r_z} \) Residual normal stress component in z-direction

\( \sigma_y \) Yield Strength

\( \tau_{r_{xy}} \) Residual shear stress component in xy-plane

\( \tau_{r_{yz}} \) Residual shear stress component in yz-plane

\( \tau_{r_{zx}} \) Residual shear stress component in zx-plane

**Abbreviation:**

ASME American Society of Mechanical Engineers

ECM Elastic Compensation Method

EPP Elastic Perfectly Plastic

IPC In-Plane Closing

IPO In-Plane Opening

LDYM Load Dependent Yielding Modification