AUTOMATIC GENERATION OF STRUT-AND-TIE MODELS USING TOPOLOGY OPTIMIZATION

By

Mohamed Hassan Mahmoud Fahmy Abdelbarr
B.Sc. Civil Engineering, Cairo University, 2010.

A Thesis submitted to the
Faculty of Engineering at Cairo University
In partial fulfillment of the
Requirements for the degree of

MASTER OF SCIENCE

In

STRUCTURAL ENGINEERING

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Chapter (1)

INTRODUCTION

1.1 General

The main objective of structures is to transfer the applied loads from their points of application to the soil in safe and economical way. There are usually multiple objective criteria that must be considered by the structural design when designing a structure. The main objectives are functionality, serviceability, strength and economy. The load transfer mechanism of structural concrete members is not the function of a single variable and it depends on the geometry, loading and support conditions of the member. The strut-and-tie method provides better understanding of the behavior of structural concrete under different conditions of loading. Strut-and-tie model is a method of design for reinforced and prestressed concrete that reduces complex states of stress within a structure to a collection of simple stress paths. Structure members are divided to B-regions where Bernoulli equation for linear strain is applicable, and D-region which stand for discontinuity regions. Strut-and-tie model is used to model the D-regions with complex stress field.

Traditional methods are not efficient in developing strut-and-tie models in structural concrete members since they involve a trial-and-error process. The strut-and-tie models obtained by trial-and-error methods vary greatly with the design past experience in order to choose an appropriate strut-and-tie model for a structural concrete members. The objective of this thesis is to illustrate an efficient method to automatic generate the truss model. The structural optimization techniques are effective tools that can be used to improve the performance of structures in terms of the material efficiency in transferring the applied loads.
1.2 Thesis organization

Following the introduction section, chapter (2) presents an overview of topology optimization of continuum structures; various methods of structural topology optimization will be illustrated, with focus on the evolutionary structural optimization (ESO). ESO based on stress level, ESO for stiffness optimization and bi-directional evolutionary structural optimization (BESO) will be discussed in details.

Chapter (3) presents an overview on the historical methods for developing strut-and-tie models, some basic definitions necessary to understand the basics of strut–and-tie model, solving the strut-and-tie model as topology optimization problem, and dimensioning strut-and-tie model using ACI 318-08.

Chapter (4) presents numerical examples of the bi-directional evolutionary structure optimization to demonstrate the effectiveness of this method to generate strut-and-tie models in reinforced concrete structures. Many case studies are presented with different geometry and loads configurations, deep beams with various span to depth ratio, deep beams with openings, continuous deep beams, and bridge pier.

Chapter (5) outlines the conclusions based on the results of case studies. Finally, some recommendations for future work are summarized at the end of the chapter.
Chapter (2)

TOPOLOGY OPTIMIZATION OF CONTINUUM STRUCTURES

2.1 Introduction

Structural optimization seeks to achieve the best performance for a structure while satisfying various constraints such as a given amount of material. Optimal structural design is becoming very important due to the limited material resources, environmental impact and technological competition, all of which demand lightweight, low-cost and high-performance structures.

Over the last three decades the availability of high-speed computers and the rapid improvements in algorithms used for design optimization have transformed the topic of structural optimization from the previous narrowness of mostly academic interest to the current stage where a growing number of engineers and architects start to experiment with and benefit from the optimization techniques. There have been more and more research and development activities directed towards making the structural optimization algorithms and software packages available to the end-users in an easy, reliable, efficient and inexpensive form.

Various structural optimization methods have been developed for the layout design of structures in the past few decades these methods can be classified into two categories, analytical methods and numerical methods. The analytical methods used the mathematical theory of calculus to study behavior of the material layouts of structural components and simple skeletal structures under simple loading conditions. Numerical methods generate optimal designs automatically in an iterative method using mathematical programming. It can solve large and complex problems.
The types of structural optimization may be classified into three categories, i.e. size, shape and topology optimization. Size optimization is to find the optimal design by changing the size variables such as the cross-sectional dimensions of trusses and frames, or the thicknesses of plates. This is the easiest and earliest approach to improving structural performance. Shape optimization is mainly performed on continuum structures by modifying the predetermined boundaries to achieve the optimal designs. Topology optimization for discrete structures, such as trusses and frames, is to search for the optimal spatial order and connectivity of the bars. Topology optimization of continuum structures is to find the optimal designs by determining the best locations and geometries of cavities in the design domains. Figure (2.1) shows the difference between the shape and topology optimization.

![Figure (2.1) Topological versus shape optimization](image)

Structural optimization is to seek the minimum (or maximum) value of a function subject to design constraints. This involves the determination of optimal design variables that minimize (or maximize) the objective function. The general formulation of an optimization problem may be expressed as
Minimize \( f(x) \)

Subject to \( g_j(x) \leq g_j^* \quad j=0,1,2,\ldots,k \) \hfill (2.1)

\[ X_i^L < X_i < X_i^u \quad i=0,1,2,\ldots,n \] \hfill (2.2)

Where \( f(x) \) is the objective function, \( g_j(x) \) is the \( j \)th constraint, \( g_j^* \) is the limit of the \( j \)th constraint, \( k \) is the number of constraints, \( X_i^L \) and \( X_i^u \) and are the lower and upper bounds on the design variable \( X_i \), and \( n \) is the total number of design variables. The objective function can be the weight or the overall stiffness of a structure. The constraints can be geometrical restrictions such as the height and width of the structure or behavioral restrictions such as stresses, displacements, mean compliance, frequency and buckling loads.

2.1.1 Structural Optimization

The shape optimization problem of continuum structures has been solved by Zienkiewicz and Campbell (1973) using a sequential programming technique. Haftka and Grandhi (1986) presented a survey on structural shape optimization in which the boundary variation method has extensively been used. The boundary variation method is implemented by using the mesh moving schemes to express the shape of a given design. The coordinates of nodal points of the finite element model were treated as design variables. Special techniques for maintaining the regularity of the finite element model are usually required to obtain an optimal shape in the boundary variation method (Kikuchi et al. 1986).

In shape optimization using the boundary variation method, the finite element model is not fixed during the optimization process so that re-meshing the model is required at each iteration. To avoid these, Bendsøe and Kikuchi (1988) proposed a homogenization-based optimization (HBO) method for topology and shape design of continuum structures using a fixed design domain. In the HBO method, topology optimization is transformed to a material redistribution problem in a
continuum design domain made by composite material with microstructures. The effective material properties of the composite material are computed using the theory of homogenization. For the maximum stiffness design, the mean compliance of a structure is used as the objective function while the constraint is imposed on the material volume. Extensive studies on the HBO method have been developed (Bendsøe 1989; Suzuki and Kikuchi 1991; Hassani and Hinton 1999; Krog and Olhoff 1999; Bendsøe and Sigmund 2003). The HBO method was extended by Diaz and Bendsøe (1992) and Bendsøe et al. (1995) to structures under multiple loading conditions. Tenek and Hagiwara (1993) applied the HBO method to the topology optimization of plates in bending.

The density-based optimization (DBO) method has been developed as an alternative approach to the minimum compliance optimization problem (Bendsøe 1989; Mlejnek and Schirrmacher 1993; Yang and Chuang 1994; Ramm et al. 1994; Sigmund 2001). In the DBO method, material properties are assumed to be constant within each finite element employed to discretize the design domain and element relative densities are treated as design variables. The effective material properties are computed by the relative material density raised to some power times the material properties of the solid material. The material model described by the power law is artificial since no such physical material exists. The power law approach must be combined with perimeter constraints, gradient constraints or filtering techniques to ensure the existence of solutions (Sigmund 2001). Gea (1996) presented a microstructure-based design domain method, which employs a closed-form expression for the effective Young’s modulus and shear modulus in terms of phase properties and volume fractions. Yang (1997) extended the DBO method to general topology optimization problems in which compliance, displacements and natural frequencies are treated as constraints. In comparison with the HBO method, the DBO approach is simple to implement and can produce conceptual designs.
The difficulty associated with continuum topology optimization is to determine an appropriate termination criterion in optimization algorithms to obtain the global optimum. For the maximum stiffness topology design problem, the mean compliance of a structure is commonly used in most of the existing optimization methods as the objective function, and the constraint is imposed on a somewhat arbitrarily specified material volume. Based on this termination criterion, using a different percentage of the material volume as the constraint results in different designs. Obviously, appropriate criteria for identifying the global optimum are still lacking in these continuum topology optimization methods that could lead to many local optima if they are not the worst or dangerous designs. To overcome this problem, Liang (2001a, b) and Liang and Steven (2002) have developed the performance based optimization (PBO) method that employs energy-based performance indices and performance-based optimality criteria (PBOC) to achieve the global optimal designs of continuum structures with mean compliance constraints. These energy-based performance indices and PBOC can be incorporated into other optimization methods to ensure a global optimal solution to be obtained. One of the most used technique in topology optimization is The evolutionary structural optimization (ESO) and its extend The bi-directional evolutionary structural optimization (BESO) that will be discussed in details in the next section.

2.2 The evolutionary structural optimization

The evolutionary structural optimization (ESO) method is one of the most popular techniques for topology optimization, The ESO method was first proposed by Xie and Steven in the early 1990s (Xie and Steven1992) and has since been continuously developed to solve a wide range of topology optimization problems (Xie and Steven 1997). ESO is based on the simple concept of gradually removing inefficient material from a structure.
2.2.1 ESO based on stress level

Ideally the stress in every part of the structure should be close to the same safe level. For that the low-stressed material is assumed to be under-utilized and is therefore removed subsequently by deleting element from the finite element model.

The stress level of each element is determined by comparing, for example, the von Mises stress of the element \( \sigma_{e}^{vm} \) with the maximum von Mises stress of the whole structure \( \sigma_{\text{max}}^{vm} \). After each finite element analysis, elements which satisfy the following condition are deleted from the model.

\[
\frac{\sigma_{e}^{vm}}{\sigma_{\text{max}}^{vm}} < RR_{i}
\]

(2.3)

Where \( RR_{i} \) is the current rejection ratio (RR).

Such a cycle of finite element analysis and element removal is repeated using the same value of \( RR_{i} \) until a steady state is reached, which means that there are no more elements being deleted using the current rejection ratio. At this stage an evolutionary rate, \( ER \), is added to the rejection ratio, i.e.

\[
RR_{i+1} = RR_{i} + ER
\]

(2.4)

With the increased rejection ratio the iteration takes place again until a new steady state is reached.

The evolutionary process continues until a desired optimum is obtained, for example, when there is no material in the final structure which has a stress level less than 25% of the maximum. The evolutionary procedure can be summarized as follows:

1- Model the structure using a fine mesh of finite elements.
2- Carry out finite element analysis for the structure.
3- Remove elements which satisfy the condition in (2.3).
4- Increase the rejection ratio according to Equation (2.4) if a steady state is reached.
5- Repeat Steps 2 to 4 until a desired optimum is obtained.

2.2.2 ESO for stiffness or displacement optimization

Stiffness is one of the important factors that must be taken into account in the design of structures as buildings and bridges. The mean compliance can be defined by the total strain energy of the structure or the external work done by applied loads as

\[ C = \frac{1}{2} f^T u \]  \hspace{1cm} (2.5)

Where \( f \) is the force vector and \( u \) is the displacement vector. In the finite element analysis, the static equilibrium equation of a structure is expressed as

\[ Ku = f \]  \hspace{1cm} (2.6)

Where \( K \) is the global stiffness matrix. When the \( i \)th element is removed from the structure, the stiffness matrix will change by

\[ \Delta K = K^* - K = -K_i \]  \hspace{1cm} (2.7)

Where \( K^* \) is the stiffness matrix of the resulting structure after the element is removed and \( K_i \) is the stiffness matrix of the \( i \)th element. It is assumed that the removal of the element has no effect on the applied load \( f \). By varying both sides of Equation (2.6) and ignoring a higher order term, the change of the displacement vector is obtained as

\[ \Delta u = -K^{-1} \Delta Ku \]  \hspace{1cm} (2.8)
From equations (2.5) and (2.8) we have

\[ \Delta C = \frac{1}{2} f^T \Delta u = \frac{1}{2} f^T K^{-1} \Delta Ku = \frac{1}{2} u_i^T K_i u_i \]  \hspace{1cm} (2.9)

Where \( u_i \) is the displacement vector of the \( i \)th element. Thus, the sensitivity number for the mean compliance can be defined as

\[ \alpha_i^e = \frac{1}{2} u_i^T K_i u_i \]  \hspace{1cm} (2.10)

The above equation indicates that the increase in the mean compliance as a result of the removal of the \( i \)th element is equal to its elemental strain energy. To minimize the mean compliance (which is equivalent to maximizing the stiffness) through the removal of elements, it is evident that the most effective way is to eliminate the elements which have the lowest values of \( \alpha_i \) so that the increase in \( C \) will be minimal. The number of elements to be removed is determined by the element removal ratio (ERR) which is defined as the ratio of the number of elements removed at each iteration to the total number of elements in the initial or the current FEA model. The evolutionary procedure for stiffness optimization is given as follows:

1- Model the structure using a fine mesh of finite elements.
2- Carry out finite element analysis for the structure.
3- Calculate the sensitivity number for each element using Equation (2.10).
4- Remove a number of elements with the lowest sensitivity numbers according to a predefined element removal ratio ERR.
5- Repeat Steps 2 to 4 until the mean compliance (or the maximum displacement, etc.) of the resulting structure reaches a prescribed limit.

There is no steady state specified in the optimization for stiffness, a sensitivity number can be derived for a displacement constraint where the maximum
displacement of a structure or the displacement at a specific location has to be within a prescribed limit.

**2.3 Bi-directional evolutionary structural optimization method**

### 2.3.1 Introduction

The bi-directional evolutionary structural optimization (BESO) method allows material to be removed and added simultaneously. The initial research on BESO was conducted by Yang et al. (1999) for stiffness optimization. In their study, the sensitivity numbers of the void elements are estimated through a linear extrapolation of the displacement field after the finite element analysis. Then, the solid elements with the lowest sensitivity numbers are removed from the structure, and the void elements with the highest sensitivity numbers are changed into solid elements. The numbers of removed and added elements in each iteration are determined by two unrelated parameters: the rejection ratio (RR) and the inclusion ratio (IR) respectively.

The BESO concept has also been applied to ‘full stress design’ by using the von Mises stress criterion (Querin et al. 2000). In their BESO algorithm, elements with the lowest von Mises stresses are removed and void elements near the highest von Mises stress regions are switched on as solid elements. Similarly, the numbers of elements to be removed and added are treated separately with a rejection ratio and an inclusion ratio respectively.

The method of removal and addition of such elements based on the removal ratio (RR) and inclusion ratio (IR) had some problems. "The user must carefully select the values of RR and IR in order to obtain a good design; otherwise, the algorithm may not produce an optimal solution (Rozvany 2009)." Another problem appears the low computational efficiency due to large number of iterations and the choice between a large numbers of topologies. In many cases the convergence of
objective function is often highly chaotic. It should be pointed that the introduction of many halls in the finite element model may cause numerical instability (Sigmund and Petersson 1998; Bendsøe and Sigmund 2003) which is called mesh-dependency.

For that a new BESO algorithm for stiffness optimization developed by (Huang and Xie 2007) which solves the related problems to topology optimization of continuum structures such as a proper statement of the optimization problem, checkerboard pattern, mesh-dependency and convergence of solution.

2.3.2 Problem statement and sensitivity number

It was clear that the objective and constraints in the original ESO/BESO methods are very vague, especially for stiffness optimization. The original ESO may lead to a worse solution if an inappropriate constraint or large element removal is applied. To avoid a worse design, the compliance volume product is used as a performance index (Liang et al. 2000) and the evolutionary procedure is stopped once the performance index drops dramatically. However, the problem may still arise. For example, the whole evolutionary procedure may stop before the required displacement constraint is satisfied.

Topology optimization is often aimed at searching for the stiffest structure with a given volume of material. In ESO/BESO methods, a structure is optimized by removing and adding elements. That is to say that, the element itself, rather than its associated physical or material parameters, is treated as the design variable. Thus, the optimization problem with the volume constraint is stated as
Minimize

\[ (2.11a) \]

Subjected to:

\[ (2.11b) \]

\[ x_i = 0 \text{ or } 1 \quad (2.11c) \]

Where \( f \) and \( u \) are the applied load and displacement vectors and \( C \) is known as the mean compliance. \( V_i \) is the volume of an individual element and \( V^* \) the prescribed total structural volume. \( N \) is the total number of elements in the system. The binary design variable \( x_i \) declares the absence (0) or presence (1) of an element.

When a solid element is removed from a structure, the change of the mean compliance or total strain energy is equal to the elemental strain energy (Chu et al. 1996). This change is defined as the elemental sensitivity number:

\[ \alpha_i^e = \Delta C_i = \frac{1}{2} u_i^T K_i u_i \quad (2.12) \]

Where \( u_i \) is the nodal displacement vector of the \( i \)th element, \( K_i \) is the elemental stiffness matrix. When a nonuniform mesh is assigned, the sensitivity number should consider the effect of the volume of the element. In such a case, the sensitivity number can be replaced with the elemental strain energy density as

\[ \alpha_i^e = e_i = \left( \frac{1}{2} u_i^T K_i u_i \right) / V_i \quad (2.13) \]
The original ESO procedure for stiffness optimization is directly driven by gradually removing elements with the lowest sensitivity numbers defined in Equation (2.12) or (2.13). The sensitivity numbers for void elements are assumed to be zero initially. The reason for this setting will be given in the next section. To add material into the design domain, a filter scheme will be used to obtain the sensitivity number for the void elements and to smooth the sensitivity number in the whole design domain. More importantly, by using the filter scheme the problems of checkerboard pattern and mesh-dependency will be resolved at once.

2.3.3 Filter scheme and improved sensitivity number

2.3.3.1 Checkerboard and mesh-dependency problems

When a continuum structure is modeled using low order bilinear (2D) or trilinear (3D) finite elements, the sensitivity numbers could become discontinuous across element boundaries. Figure (2.2) shows a typical checkerboard pattern of a continuum structure from the original ESO method. The presence of checkerboard pattern causes difficulty in determining the optimal structure. To eliminate the formation of checkerboard patterns in the ESO method, a simple scheme of averaging the sensitivity numbers of neighboring elements has been presented by Li et al. (2001). However, this smoothing algorithm cannot overcome the mesh-dependency problem.
2.3.3.2 Filter scheme for BESO method

Before applying the filter scheme, nodal sensitivity numbers are defined by averaging the elemental sensitivity numbers as follows:

\[ \alpha_j^n = \sum_{i=1}^{M} \omega_i \alpha_i^e \quad (2.14) \]

Where \( M \) denotes the total number of elements connected to the \( j \)-th node. \( \omega_i \) is the weight factor of the \( i \)-th element and \( \sum_{i=1}^{M} \omega_i = 1 \). \( \omega_i \) can be defined by

\[ \omega_i = \frac{1}{M-1} \left( 1 - \frac{r_{ij}}{\sum_{i=1}^{M} r_{ij}} \right) \quad (2.15) \]

Where \( r_{ij} \) is the distance between the centre of the \( i \)-th element and the \( j \)-th node. The above weight factor indicates that the elemental sensitivity number has larger effect on the nodal sensitivity number when it is closer to the node.

The above nodal sensitivity numbers will then be converted into elemental sensitivity numbers. A filter scheme is used to carry out this process. The filter has a length \( r_{\text{min}} \) that does not change with mesh refinement. The primary role of the scale parameter \( r_{\text{min}} \) in the filter scheme is to identify the nodes that will influence
the sensitivity of the ith element. This can be determined by drawing a circle of radius \( r_{\text{min}} \) centered at the centroid of ith element, thus generating the circular sub-domain \( \Omega_i \) as shown in Figure 2.3. Usually the value of \( r_{\text{min}} \) should be big enough so that \( \Omega_i \) covers more than one element. The size of the sub-domain \( \Omega_i \) does not change with mesh size. Nodes located inside \( \Omega_i \) contribute in the computation of the improved sensitivity number of the ith element as

\[
\alpha_i = \frac{\sum_{j=1}^{K} \omega(r_{ij}) \alpha_j}{\sum_{j=1}^{K} \omega(r_{ij})}
\]  

(2.16)

Where \( K \) is the total number of nodes in the sub-domain \( \Omega_i \), \( \omega(r_{ij}) \) is the linear weight factor defined as

\[
\omega(r_{ij}) = r_{\text{min}} - r_{ij} \quad (j = 1, 2, 3, \ldots, K)
\]  

(2.17)

Figure 2.3 Nodes located inside the circular sub-domain \( \Omega_i \) are used in the filter scheme for the ith element. [6]

It can be seen that the filter scheme smoothes the sensitivity numbers in the whole design domain. Thus, the sensitivity numbers for void elements are automatically obtained. They may have high values due to high sensitivity numbers of solid elements within the sub-domain \( \Omega_i \). Therefore, some of the void elements may be
changed to solid elements in the next iteration. By adopting this simple technique, many numerical problems in topology optimization, such as checkerboard and mesh-dependency, can be effectively overcome. The filter scheme requires little extra computational time and is very easy to implement in the optimization algorithm.

2.3.3.3 Stabilizing the evolutionary process

As discussed in the previous section the filter scheme can effectively solved the mesh-dependency problem. However, the objective function and the corresponding topology may not converge. Huang and Xie (2007) has found that the averaging the sensitivity number is ineffective way to solve this problem. The simple averaging scheme is given as

\[ \alpha_i = \frac{\alpha_i^k + \alpha_i^{k-1}}{2} \]  

(2.18)

Where k is the current iteration number. Figure 2.4(b) shows the evolution history obtained by adopting the stabilization scheme defined in equation (2.18). Compared to the result in Figure 2.4(a), the new solution is highly stable in both the topology and the objective function (the mean compliance) after the constraint volume fraction (50 %) is achieved.