Heat-Exchanger Networks Synthesis with Fuzzy Inlet Temperatures Using Linear Membership Function and Analytic Hierarchy Process

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Abstract: Heat exchanger network synthesis with uncertain source-stream temperatures has been formulated as a fuzzy multi-objective mixed-integer nonlinear programming problem (F-MO-MINLP) to handle the multiple objectives of minimum utility consumption, minimum total area, and even minimum number of matches. Inlet Temperatures are proposed as fuzzy variables. The linear (Triangular) membership function has been employed to describe this variation of Inlet Temperatures at different a-cut values. The effect of variation of the a-cut values on the various objectives has been investigated. The Analytic Hierarchy Process (AHP) has been utilized to obtain the weights of objectives. Two examples are presented to demonstrate the various aspects dealt in the work.

Keywords: Heat Exchanger Network Synthesis, Multi-objective Mixed Integer Nonlinear programming problem, Membership function, a-cut Based Method and Fuzzy Inlet Temperatures and Analytic Hierarchy Process.

I. INTRODUCTION

In the Chemical and Petrochemical Industry, energy conservation has become one of the most relevant current concerns. Heat exchanger network (HEN) is taken as one of the most important measures for energy saving in process industry. A HEN synthesis problem can be described as the one that synthesize a HEN configuration to reach some assigned targets, with given heating/cooling utilities and hot/cold process streams to be cooled/heated from nominal inlet temperatures to specified target temperatures. Heat exchanger network’s ability to meet the altering process conditions is an important design consideration in addition to the network economics. In the actual operation, various operating parameters, such as stream temperature, crude oil density, process stream flow rates and heat transfer coefficient of heat exchangers may deviate from the nominal value. Furthermore, the operating conditions may alter at different operation periods. To ensure feasible operation and avoid deprivation of the system economics, flexible design of heat exchanger network is essential [1]. Xiao et al. [2] presented a novel methodology for simultaneously optimizing synthesis and cleaning schedule of flexible heat exchanger network (HEN) to decrease the total annual cost (TAC) and further improve network flexibility. Osman et al. [3] proposed minor retrofit using the available utility paths in the HEN while considering the process streams’ temperature flexibility (TF) to reduce the process consumption in the HEN. Escobar et al. [4] presented a computational framework for automatically generating flexible Heat Exchanger Networks (HENs) over a specified range of expected variations in the inlet temperatures and flow rates of the process streams. Zhang et al. [5] formulated the flexible synthesis problem for heat exchanger network (HEN) to a mixed integer nonlinear program (MINLP) model to satisfy flexible criterion during input span. Habbi et al. [6] addressed the problem of fuzzy identification of a pilot heat exchanger using fuzzy clustering algorithm. Ahmad et al. [7] focuses on heat exchanger network design for multi-period operation wherein the operating conditions of a process may vary with time using simulated annealing. Osman et al. [8] presented Fuzzy Analytic Hierarchy Process to Synthesis Heat-Exchanger Networks with Fuzzy Heat Capacity Flow Rates. EL-Tentamy and Gabr [9] discussed the design of optimal flexible heat exchanger networks that adapt with changes in streams’ start and target temperatures and heat capacity flow rates. Bakhtiari and Bedard [10] modified the network pinch approach to consider the flexibility in the supply and the target temperatures of the process streams. Bakar et al. [11] developed a new model-based integrated process design and control (IPDC) methodology, which includes cost optimality and controllability aspects at the early HEN design stage. Jiang and Chang [12] first constructed and solved a single-period model to produce the optimal design for each period individually. A timesharing strategy is then applied to integrate all such single-period designs so as to reduce the overall capital investment while still keeping the utility consumption rates in every period at the minimum levels. Zheng et al. [13] proposed an approach for flexible HEN synthesis under severe operation uncertainty, which is represented by the Probability bounds analysis (PBA) theory and is sampled by a double-loop sampling method. The presented method by Li et al. [14], for flexible heat exchanger network synthesis, is sequentially implemented by two main steps: initially structure synthesis at the nominal operating point and an iterative approach with strong robustness for area optimization. Zhou et al. [15] used a multi-level, multi-factor and Non-structural fuzzy decision making method (NSFDMM) in the optimal selection of compact heat to develop the energy saving technology for the design of the optimal heat exchangers with the best comprehensive performance adapted to the different working conditions.
In multi-objective optimization problems, the objective functions usually conflict with each other. Thus it is impossible to attain their optima, simultaneously. Therefore, the decision maker (DM) must make some compromise among these objectives. The weighting-sum method is the one that is used most often. Basing on the subjective comprehension for each objective, the DM of this method can weigh and sum up these objectives into a scalar form, and then find the solution by any existing single-objective optimization method. In this work, the Analytic Hierarchy Process (AHP) has been used to assist the DM in choosing the "best" alternative. The Triangular Membership function had been employed for defuzzification of the fuzzy Inlet Temperatures to obtain the single objective nonfuzzy function to be solved employing GAMS program.

In this paper, the various aspects of Multi-objective MINLP Problems with Fuzzy Parameter in constraints is reviewed in Section II. In Section III, Analytic Hierarchy Process (AHP) is explained. The Fuzzy Heat Exchanger network synthesis (F-HEN) problem is defined and problem formulation is depicted in Section IV. Solution of the formulated problem and Algorithm (F-HEN and AHP) is presented in Section V. To verify these aspects two examples are presented in Section VI. Statistical analysis of the results is presented in Section VII. Finally we conclude the paper in Section VIII.

II. MULTI-OBJECTIVE NLP PROBLEMS WITH FUZZY PARAMETER IN CONSTRAINTS

Let us consider the following Multi-objective Nonlinear Programming Problems with fuzzy parameter in constraints (FMONLP).

\[ \min f(x) = (f_1(x), f_2(x), ..., f_k(x))^T \]

subject to

\[ S = \left\{ x \in \mathbb{R}^n \mid g_r(x, \tilde{c}_r) \leq 0, r = 1, 2, ..., m \right\} \]

Where \( x \) is an \( n \)-dimensional vector of decision variables, \( S \subset \mathbb{R}^n \) is the feasible set of decision variables, and \( f_i(x), i = 1, 2, ..., k \) are real valued functions of \( x \in \mathbb{R}^n \). \( \tilde{c}_r = (\tilde{c}_r^1, \tilde{c}_r^2, ..., \tilde{c}_r^g) \) represent vector of fuzzy parameters in the constraint functions. \( S \) is a non empty set. These fuzzy numbers are assumed to be characterized as the fuzzy numbers introduced in [16].

**Triangle linear membership function** depicted in Fig. 1, and defined by the real fuzzy numbers \( \tilde{v} \) forms a convex continuous fuzzy subset of the real line whose membership function \( \mu_{\tilde{v}}(v) \) is:

1. a continuous mapping from \( R \) to the closed interval \([0, 1]\);
2. \( \mu_{\tilde{v}}(v) = 0 \) for all \( v \in (-\infty, v_1] \);
3. strictly increasing on \( v \in [v_1, v_2] \); (2)
4. \( \mu_{\tilde{v}}(v) = 1 \) for all \( v = v_2 \);
5. strictly decreasing on \( v \in [v_2, v_3] \);
6. \( \mu_{\tilde{v}}(v) = 0 \) for all \( v \in [v_3, \infty) \).

It must be noted that the membership function \( \mu_{\tilde{v}}(v) \) is a function of \( v \), \( i = 1, 2, 3 \) at a certain level \( \alpha \).

**Definition:** The \( \alpha \) level set of the fuzzy numbers \( \tilde{c} \) is defined as the ordinary set \( L_\alpha(\tilde{c}) \) for which the degree of their membership functions exceeds the level \( \alpha \in [0, 1] \).

\[ L_\alpha(\tilde{c}) = \left\{ (c) / \mu_{\tilde{c}}(c) \geq \alpha, \quad i = 1, 2, ..., m \right\} \]

For a certain degree \( \alpha \), the (FMONLP) problem can be understood as the following nonfuzzy \( \alpha \) multi-objective nonlinear programming problem (\( \alpha \)MONLP) [17].

\[ \min f(x) = (f_1(x), f_2(x), ..., f_k(x))^T \]

subject to

\[ S = \left\{ x \in \mathbb{R}^n \mid g_r(x, \tilde{c}_r) \leq 0, r = 1, 2, ..., m \right\} \]

\( \epsilon \in L_\alpha(\tilde{c}) \)

III. APPLICATION OF THE ANALYTIC HIERARCHY PROCESS

A. Analytical Hierarchy Process

Analytical hierarchy process (AHP) is a systematic procedure developed by [18] for representing the elements of any problem hierarchically [19] and is designed to solve complex problems involving multiple criteria. The AHP is a decision-aid that can provide the decision maker (DM) with relevant information to assist the DM in choosing the “best” alternative or to rank a set of alternatives [20].

B. The AHP process steps

The AHP utilizes pair wise comparisons to a numerical representation for the relationship between two elements that share a common parent in the hierarchy. The formulated matrix shows results of all such comparisons. AHP employs an underlying scale with values from 1 to 9 to rate the relative preferences for the two items (two objectives \( f_i \) and \( f_j \)).

Saaty, T. L. [21] shows in Table I the numerical ratings recommended for the verbal preferences expressed by the
Computing the vector of objective weights

The pair-wise comparison matrix \( A(n \times n) \) has the following criteria:

Each entry \( a_{jk} \) of \( A \) represents the importance of criterion \( j \) relative to criterion \( k \). The element 

\[ a_{kj} = \frac{1}{a_{jk}} \quad \forall \; k \neq j \]

and the element \( a_{ii} = 1 \) \( \forall \; i \)

Finally, the objective weight vector \( w \) is built by averaging the entries on each row of \( A \) norm (if the matrix is consistent). For the consistent matrices, every two elements must satisfy

\[ a_{ik} a_{kj} = a_{ij} \].

Consistent matrix \( A \) has rank 1 and one Eigenvalue with value \( n \).

There is an eigenvalue/eigenvector method and geometric mean method to get the weights for inconsistent matrices. A method to measure the degree of consistency among the pair wise judgments provided by the decision-maker is needed. Compute the Consistency Index which is defined as follows:

\[ CI = (\lambda_{\text{max}} - n)/(n - 1) \]

Compute the Consistency Ratio (CR) which is defined as:

\[ CR = CI / RI \]

Where RI, the Random index, is the consistency index of a randomly generated pair-wise comparison matrix. It can be shown that RI depends on the number of elements being compared as shown in Table II, [21]. Algorithm 1 depicts the various steps for computing the vector of objective weights.

### Table I

**DEFINITION OF THE COMPARISONS**

<table>
<thead>
<tr>
<th>Verbal Preference</th>
<th>Numerical Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equally</td>
<td>1</td>
</tr>
<tr>
<td>Equally to moderately</td>
<td>2</td>
</tr>
<tr>
<td>Moderately</td>
<td>3</td>
</tr>
<tr>
<td>Moderately to strongly</td>
<td>4</td>
</tr>
<tr>
<td>Strongly</td>
<td>5</td>
</tr>
<tr>
<td>Strongly to very strongly</td>
<td>6</td>
</tr>
<tr>
<td>Very strongly</td>
<td>7</td>
</tr>
<tr>
<td>Very strongly to extremely</td>
<td>8</td>
</tr>
<tr>
<td>Extremely preferred</td>
<td>9</td>
</tr>
</tbody>
</table>

The pair-wise comparison matrix \( A(n \times n) \)

\[ A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \]

\[ f_1 f_2 f_n \]

Algorithm 1: (Computing the vector of objective weights)

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1:</td>
<td>Establish the pair wise comparison matrix ( A ) via pair wise relative preference of every two objective functions using Table I</td>
</tr>
<tr>
<td>Step 2:</td>
<td>If matrix ( A ) is not consistent go to step 6</td>
</tr>
<tr>
<td>Step 3:</td>
<td>Compute eigenvector</td>
</tr>
<tr>
<td>Step 4:</td>
<td>Normalize eigenvector, to get the objective weights</td>
</tr>
<tr>
<td>Step 5:</td>
<td>Stop</td>
</tr>
<tr>
<td>Step 6:</td>
<td>Measure the Consistency Ratio using (5) and Table II.</td>
</tr>
</tbody>
</table>

If \( CR < 0.1 \) go to step 3.

Else ask DM to change the pair wise matrix \( A \) and go to Step 2

### IV. FUZZY-HEN PROBLEM

A. Heat Exchanger Network Synthesis (HENS)

Heat exchanger network synthesis (HENS) has been one of the most well studied issues within process synthesis during the last three decades [22]. In the heat recovery system, the process streams exchange heat so as to reduce the hot and cold utility requirements. The only units in a heat recovery system are the heat exchangers.

A heat exchanger network synthesis (HENS) problem can be described as the one that synthesize a HEN configuration to reach some assigned targets such as minimum utility consumption, minimum total number of heat exchangers, minimum area etc., with given heating/cooling utilities and hot/cold process streams be cooled / heated from nominal inlet temperatures to specified target temperatures [23].

B. Fuzzy Inlet Temperatures and membership functions:

The Fuzzy optimization and synthesis of chemical processes denote the use of specific methods and strategies in those cases where the values of some parameters are not known exactly but are given by Fuzzy set theory models instead. Fuzzy set theory is used to measure nonrandom processes and can be used to quantify the confidence of a process design that is subject to fuzzy uncertainties. For a HEN, variations in the inlet temperatures are the most significant variations with a high potential of occurrence. The potential of such variations to occur can be related to a membership function (MF). Zadeh [24] first proposed this cognitive concept. The shape of the MF depends on the way a design engineer or operations engineer conceptualizes the expected changes in the operating conditions. In this work, we deal with the variation of the inlet temperatures employing triangle membership function depicted in Fig. 1.

C. Problem statement

In order to address the heat integration problem, it will be assumed that given are a set of hot process streams \( HP \) to be cooled and a set of cold process streams \( CP \) to be heated. Specified are also a set of hot utilities \( HU \) and a set of cold utilities \( CU \) and their corresponding temperatures. The solution defines the network by providing the following [25]:

### Table II

**RANDOM CONSISTENCY**

<table>
<thead>
<tr>
<th>( n )</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( RI )</td>
<td>0.58</td>
<td>0.9</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
</tr>
</tbody>
</table>
1. Utilities required.
2. Stream matches and the number of units.
3. Heat loads and operating temperatures of each exchanger.
4. Network configuration and flows for all branches.
5. Area of each exchanger.

D. Formulation of Fuzzy Heat Exchanger Network (FHEN) statement:

The multi-criteria optimization approach will be adopted for FHEN synthesis and the problem is formulated as a multi-objective mixed-integer nonlinear programming (MOMINLP). The fuzzy inlet temperatures \( \overline{T}_{i}^{in} \) are proposed as fuzzy variables not parameters. The fuzzy fuzzy inlet temperatures \((T_{inj}^{in} - d_{j}, T_{inj}^{in} + d_{j})\) are of ranges \( T_{inj}^{in} \) in s, s_1 \in [-2,2] CPj and \( [T_{inj}^{in} - d_{j}, T_{inj}^{in} + d_{j}] \). \( d_{i} \) and \( d_{j} \) are fixed values for every \( i \) hot and \( j \) cold streams.

- **Multi-Objective Functions for FHEN**

The objective functions are minimizing the exchangers’ areas \( f_{1} \), minimizing the number of matches \( f_{2} \) and minimizing utility \( f_{3} \) which can be simultaneously considered as conflicting objectives for synthesis of the network structure. The symbols that appear in the following equations are explained in the Nomenclature. The objective functions could be stated as follows:

\[
f_{1} = \min \sum_{i \in HP} \sum_{j \in CP} \left( q_{i,j,s} \left( \overline{U}_{i,j}^{AMTD} t_{i,j,s} \right) \right)
\]

\[
+ \sum_{i \in HC} \left( q_{cu,i} \left( \overline{U}_{cu}^{AMTD} t_{in} \right) \right)
\]

\[
f_{2} = \min \sum_{i \in HP} \sum_{j \in CP} \left( q_{hu,j} \right)
\]

\[
f_{3} = \min \sum_{i \in HP} q_{hu,j}
\]

- **The Constraints for HEN**

**Overall heat balance for each stream**

\( \left( T_{in}^{in} - T_{out}^{out} \right) FCp_{i} = \sum_{s \in ST} \sum_{j \in CP} q_{i,j,s} + q_{cu,i}, i \in HP \)

\( T_{out}^{out} = T_{in}^{in} - \overline{T}_{i}^{in}, i \in HP \) \hspace{1cm} \( T_{in}^{in} = T_{out}^{out} - \overline{T}_{i}^{out}, i \in HP \)

**Heat balance at each stage**

\( q_{i,j,s} + q_{hu,j}, j \in CP \)

\( q_{i,j,s} + q_{hu,j}, j \in CP \)

\( q_{i,j,s} + q_{hu,j}, j \in CP \)

**Assignment of superstructure inlet temperatures**

\( t_{i,j} = T_{i,j}^{in}, i \in HP, j \leq NOS + 1 \) \hspace{1cm} \( t_{i,j} = T_{i,j}^{out}, i \in HP, j > NOS + 1 \)

**Feasibility of temperature**

\( t_{i,j} \leq \overline{T}_{i,j}, i \in HP, j \leq NOS + 1, j \leq j_{max} \)

\( t_{i,j} \geq \overline{T}_{i,j}, i \in HP, j \leq NOS + 1, j \leq j_{max} \)

\( T_{i,j}^{out} \leq \overline{T}_{i,j}, i \in HP, j \leq NOS + 1, j \leq j_{max} \)

\( T_{i,j}^{out} \geq \overline{T}_{i,j}, i \in HP, j \leq NOS + 1, j \leq j_{max} \)

**Hot and cold utility load**

\( (t_{i,NOS} + 1 - T_{i,out}^{out}) FCp_{i} = q_{cu,i}, i \in HP \)

\( T_{j,out}^{out} - t_{j,1} FCp_{j} = q_{hu,j}, j \in CP \)

**Logical constraints**

\( q_{i,j,s} - \Omega_{i,j} z_{i,j,s} \leq 0, i \in HP, j \in CP, s \in ST \)

\( q_{cu,j} - \Omega_{j} z_{cu,j} \leq 0, i \in HP \)

\( q_{hu,j} - \Omega_{j} z_{hu,j} \leq 0, j \in CP \)

\( z_{i,j,s}, z_{cu,j}, z_{hu,j} = \{0,1\}, i \in HP, j \in CP, s \in ST \)

**Calculation of approach temperatures**

\( dt_{i,j,s} \leq t_{i,s} - t_{j,s} + \Gamma_{i,j} (1 - z_{i,j,s}), i \in HP, j \in CP, s \in ST \)

\( d_{cu} \leq t_{i,NOS} + 1 - T_{cu,out} + \Gamma (1 - z_{cu,i}), i \in HP \)

\( d_{hu} \leq T_{hu,out} - t_{j,1} + \Gamma (1 - z_{hu,j}), j \in CP \)

\( \Omega_{i} = FCp_{i} (T_{i,in} - T_{i,out}), i \in HP \)

\( \Omega_{j} = FCp_{j} (T_{j,out} - T_{j,in}), j \in CP \)

\( \Omega_{i,j} = \min \Omega_{i}, \Omega_{j}, i \in HP, j \in CP \)

\( \min z_{cu,j}, z_{hu,j} = \{0,1\} \)

\( \Omega_{i} = FCp_{i} (T_{i,out} - T_{i,in}) \)

\( \Omega_{j} = FCp_{j} (T_{j,in} - T_{j,out}) \)

\( \min z_{cu,j}, z_{hu,j} = \{0,1\} \)

**Where** \( f_{1}(x) = \) overall total area.

\( f_{2}(x) = \) cooler, heater and heat exchanger units, and

\( f_{3}(x) = \) Hot and cold utility loads

\( \min \sum_{i \in HP} \sum_{j \in CP} q_{cu,i} \)

\( \min \sum_{i \in HP} q_{hu,j} \)

\( \min \sum_{i \in HP} q_{hu,j} \)

\( \min \sum_{i \in HP} q_{hu,j} \)

V. Solution of the Formulated Problem

Equations (7) - (18) are used to formulate the problem and for every \( \alpha \)-cut the following algorithm 2 is employed to get the optimized solution in addition to the ideal solution and anti-ideal solution for the same \( \alpha \)-cut utilizing the GAMS program for solving the single objective nonfuzzy function.
Algorithm 2 F-HEN and AHP

Step 1: Determine the vector of objective weights from algorithm 1 using AHP.
Step 2: Determine the limits of every fuzzy variable using Membership Function based on the specified \(\alpha \)-cut.
Step 3: Formulate the single objective nonfuzzy function 
\[
(a - HEN) \cdot \text{Min} F = \sum_{k=1}^{n} w_k f_k.
\]
Step 4: Solve the \(\alpha \) mixed integer nonlinear programming problem (\(\alpha\)MINLP) of HEN problem or \((a - HEN)\) employing GAMS program. The solution gives value of inlet temperatures.
Step 5: For the same specified \(\alpha \)-cut solve the crisp problems 
\[
\text{Min} f_k(x) \quad \text{and} \quad \text{Max} f_k(x)
\]
for every \(x \in S\), totally 
\[
\alpha \leftarrow \alpha
\]
objective as single objective function employing the determined inlet temperatures, where:
\[
\text{Min} f_k(x) = f_k^\text{min} : \text{Ideal solution of } f_k(x), \text{totally} \\
\alpha \leftarrow \alpha
\]
acceptable value at \(\alpha\),
\[
\text{Max} f_k(x) = f_k^\text{max} : \text{Anti-ideal solution of } f_k(x)
\]
unacceptable value at \(\alpha\).
Step 6: Compare every objective function \(f_k(x)\) with the best \(f_k^\text{min}\) and the worst \(f_k^\text{max}\) at the chosen value of \(\alpha\)-cut.
Step 7: Repeat the above steps for the other values of \(\alpha \)-cut.
Step 8: Stop.

VI. APPLICATIONS

- Numerical Example F-HEN 1:

Consider the following 2-hot/2-cold streams example of F-HEN, with problem data presented in Table 3. Cost of Heat Exchangers and coolers (\(\$/\text{yr}^{-1}\)) = 6250 + 83.26* [Area (m\(^2\))] and Cost of Heaters (\(\$/\text{yr}^{-1}\)) = 6250 + 99.91 *[Area (m\(^2\))] [29]. The optimal solution of the HEN problem or \((a - HEN)\) gives value of inlet temperatures.

The fuzzy inlet temperatures (\(T_{in}\) ), Table IV, are:
\[
385 \leq T_{1, in} \leq 405, \\
395 \leq T_{2, in} \leq 415, \\
283 \leq T_{3, in} \leq 303, \\
343 \leq T_{4, in} \leq 363
\]

The DM gave the following preferences:
Preference of \(f_2\) than \(f_1\) is 3, Preference of \(f_2\) than \(f_3\) is 3 and Preference of \(f_1\) than \(f_3\) is 1

Table V presents the results of solution of the fuzzy problem of the example 1 at different \(\alpha \)-cut values. \(f_1(x), f_2(x)\) and \(f_3(x)\) are area, units and utility of the solution of objective functions respectively at different values of \(\alpha \)-cut. We note that the minimum value of cost is at \(\alpha_f = 0.1\). Fig. 3 depicts this solution.

\[
f_1(x) = 27043 \text{ m}^2, \\
f_2(x) = 5 \text{ units}, \\
f_3(x) = 752 \text{kW},
\]

Total Cost = 132971.2 \$

Figs. 4, 5 and 6 present the effect of varying \(\alpha \)-cut on the minimum; maximum and optimized values of objective functions of area; number of units and utility respectively. Fig. 7 shows the effect of varying \(\alpha \)-cut on the total cost of the optimized objective functions. The trend is the increase of total cost with the increase of \(\alpha \)-cut.
Fig. 2. Triangle membership functions for inlet temperatures of Example 1.

Fig. 3. Solution F-inlet Temp, Linear MF at α = 0.1 using AHP for Example 1.

Fig. 4. Relation between value of α-cut and area for Example 1.

Fig. 5. Relation between value of α-cut and No. of units for Example 1.

Fig. 6. Relation between value of α-cut and utility for Example 1.

Fig. 7. Relation between value of α-cut and cost for Example 1.

- **Numerical Example F-HEN2:**
  Consider the following 3-hot/2-cold streams example of F-HEN, with problem data presented in Table VI.
  Synthesis the HEN for the example in Table VI at $\Delta T_{\text{min}} = 30$, the fuzzy inlet temperatures ($T_{\text{in}}$) are:
  
  \[
  149 \leq T_{1,\text{in}} \leq 169, \quad 257 \leq T_{2,\text{in}} \leq 277, \quad 333 \leq T_{3,\text{in}} \leq 353 \quad \text{around} \quad 267 \]
  
  \[
  21 \leq T_{4,\text{in}} \leq 31, \quad 108 \leq T_{5,\text{in}} \leq 128 \quad \text{around} \quad 128
  \]

  Cost of Heat Exchangers, Heaters and coolers are the same as in example 1. The limits of inlet temperatures are presented in Table VII.

  **The solution**
  Algorithm 2 has been applied employing the same weights of example 1. Fig. 8 shows the Triangle membership functions for inlet temperatures.
  The limits of every fuzzy variable using linear triangle Membership function at $\alpha = 0.2$ are:
  
  \[
  151 \leq T_{1,\text{in}} \leq 167, \quad 249 \leq T_{2,\text{in}} \leq 275, \quad 335 \leq T_{3,\text{in}} \leq 351 \\
  22 \leq T_{4,\text{in}} \leq 30, \quad 110 \leq T_{5,\text{in}} \leq 126
  \]

  Table VIII presents the results of solution of the fuzzy problem of the example 2 at different $\alpha$-cut values.

  From Table VIII the minimum value of cost is at $\alpha = 0.2$. 

\[ f_1(x) = 259.08n^2, \quad f_2(x) = 8 \text{ units,} \quad f_3(x) = 2959.3kW, \]

**Total Cost = 310182.6kW**

Fig. 9 illustrates this solution. Figs. 10, 11 and 12 demonstrate the effect of varying of \( \alpha - cut \) on the minimum; maximum and optimized values of objective functions of area; number of units and utility respectively. Fig. 13 shows the effect of varying of \( \alpha - cut \) on the total cost of the optimized objective functions. Also the trend is the increase of total cost with the increase of \( \alpha - cut \).

![Triangle membership functions for inlet temperature of HEN2](image)

**Fig. 8.** Triangle membership functions for inlet temperature of Example 2.

![Solution F- inlet Temp, Linear MF at alfa=0.2 and AHP for Example2.](image)

**Fig. 9.** Solution F- inlet Temp, Linear MF at alfa=0.2 and AHP for Example 2.

### TABLE V

**THE RESULTS OF SOLUTION OF FUZZY INLET TEMPERATURES OF EXAMPLE 1 USING DIFFERENT \( \alpha \) CUTS**

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \alpha_{\min} )</th>
<th>( \alpha_{\max} )</th>
<th>units</th>
<th>utility</th>
<th>Cost $</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>77.7</td>
<td>274.32</td>
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<td>6</td>
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<td>79.83</td>
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<td>646.17</td>
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<td>642.67</td>
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<td>633.9</td>
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<td>6</td>
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<tr>
<td>0.6</td>
<td>80.29</td>
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<td>625.14</td>
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<td>7</td>
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<td>7</td>
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<td>81.95</td>
<td>272.23</td>
<td>591.63</td>
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<td>8</td>
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</tbody>
</table>

### TABLE VI

**DATA OF EXAMPLE 2.**

<table>
<thead>
<tr>
<th>index</th>
<th>Stream</th>
<th>( \tilde{T}_{IN} ) °C</th>
<th>( \tilde{T}_{OUT} ) °C</th>
<th>( F_{CP} ) (kW/Cm²)</th>
<th>( h ) (kW/Cm²°C)</th>
<th>Cost ($kW^{-1}yr^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H1</td>
<td>( T_{1,in} )</td>
<td>77</td>
<td>22.85</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>H2</td>
<td>( T_{2,in} )</td>
<td>80</td>
<td>2.04</td>
<td>0.4</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>H3</td>
<td>( T_{3,in} )</td>
<td>90</td>
<td>5.38</td>
<td>5.0</td>
<td>-</td>
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<tr>
<td>4</td>
<td>C1</td>
<td>( T_{2,in} )</td>
<td>127</td>
<td>9.33</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>C2</td>
<td>( T_{2,in} )</td>
<td>265</td>
<td>19.61</td>
<td>5.0</td>
<td>-</td>
</tr>
<tr>
<td>( hu )</td>
<td>S1</td>
<td>300</td>
<td>300</td>
<td>-</td>
<td>0.5</td>
<td>140.2</td>
</tr>
<tr>
<td>( eu )</td>
<td>W1</td>
<td>20</td>
<td>60</td>
<td>-</td>
<td>2.0</td>
<td>7.1</td>
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</table>

### TABLE VI

**THE RANGE OF THE INLET TEMPERATURES OF EXAMPLE 2.**

<table>
<thead>
<tr>
<th>Inlet Temperatures</th>
<th>L</th>
<th>M</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{1,in} )</td>
<td>149</td>
<td>159</td>
<td>169</td>
</tr>
<tr>
<td>( T_{2,in} )</td>
<td>257</td>
<td>267</td>
<td>277</td>
</tr>
<tr>
<td>( T_{3,in} )</td>
<td>333</td>
<td>343</td>
<td>353</td>
</tr>
<tr>
<td>( T_{4,in} )</td>
<td>21</td>
<td>26</td>
<td>31</td>
</tr>
<tr>
<td>( T_{5,in} )</td>
<td>98</td>
<td>118</td>
<td>138</td>
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</tbody>
</table>
TABLE VIII
THE RESULTS OF SOLUTION OF FUZZY INLET TEMPERATURES OF EXAMPLE 2 USING DIFFERENT $\alpha$ CUTS

<table>
<thead>
<tr>
<th>(\alpha_j)</th>
<th>AREA</th>
<th>UNITS</th>
<th>UTILITY</th>
<th>COST $</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>0</td>
<td>161.34</td>
<td>267.01</td>
<td>653.95</td>
<td>5</td>
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<td>0.1</td>
<td>162.72</td>
<td>260.82</td>
<td>652.88</td>
<td>5</td>
</tr>
<tr>
<td>0.2</td>
<td>162.52</td>
<td>259.08</td>
<td>650.24</td>
<td>5</td>
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<tr>
<td>0.3</td>
<td>158.33</td>
<td>213.75</td>
<td>629.39</td>
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</tr>
<tr>
<td>0.36</td>
<td>150.51</td>
<td>267.13</td>
<td>636.99</td>
<td>5</td>
</tr>
<tr>
<td>0.4</td>
<td>157.72</td>
<td>259.46</td>
<td>643.5</td>
<td>5</td>
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<td>0.5</td>
<td>157.38</td>
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<td>160.44</td>
<td>266.44</td>
<td>633</td>
<td>5</td>
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<tr>
<td>0.64</td>
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<td>265.96</td>
<td>632.23</td>
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<td>0.7</td>
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<td>260.99</td>
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<td>5</td>
</tr>
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<td>0.8</td>
<td>156.01</td>
<td>207.67</td>
<td>625.32</td>
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<td>0.9</td>
<td>159.59</td>
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<td>623.69</td>
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</tbody>
</table>

Fig. 10. Relation between value of $\alpha$-cut and area for Example 2.

Fig. 11. Relation between value of $\alpha$-cut and unit for Example 2.

Fig. 12. Relation between value of $\alpha$-cut and utility for Example 2.

Fig. 13. Relation between value of $\alpha$-cut and cost $ for Example 2.
VII. STATISTICAL ANALYSIS

To investigate the effect of variation of $\alpha$-cut on the various studied objective functions a linear regression has been performed employing the equation: objective function $= \text{slope} \cdot \alpha + \text{intercept}$. The results are presented in Table XI.

As for the area there is no consistent trend where we find in Example 2 that each of the Minimum, Maximum and Optimized area decreased with the increase of $\alpha$-cut. On the other side in Ex1 the minimum and optimized area increased while the maximum area decreased with the increase of $\alpha$-cut. The variation of $\alpha$-cut has no effect on either the minimum or maximum number of units. The maximum number of units is evaluated assuming that there is an exchange of heat between every cold and hot stream in every stage beside the possibility of exchange of heat between every stream and the utility at the outlet of the system to attain the target temperature. As for the optimized object the $\alpha$-cut has a weak effect on the number of units (increasing as in Example 1; Fig. 5 and decreasing as in Example 2; Fig. 11).

Both the minimum and Optimized Utility increased with the increase of $\alpha$-cut due to the decrease of the range of temperature to choose from while the Maximum Utility decreased with the increase of $\alpha$-cut.

The optimized total cost increases with the increase of $\alpha$-cut due to the decrease of choice of the range of temperature to choose from.

VIII. CONCLUSIONS

- The AHP helps in the decision-making process. The AHP incorporates DM’s inputs and defines a process for computing the vector of objective weights using AHP.
- Process synthesis problems under uncertainty have been mathematically represented as Fuzzy Multi-objective Mixed-Integer Nonlinear Programming (F-MO-MINLP) models. In this paper, the F-MO-MINLP problem was modified to single F-MINLP using AHP and then nonfuzzy it to the $\alpha$-MINLP form to be solved as a single MINLP utilizing the GAMS program. The solution gets the values of the inlet temperatures within the flexible range.
- Statistical analysis has been applied to investigate the effect of variation of $\alpha$-cut on the various studied objective functions.
- Variation of $\alpha$-cut has no consistent trend on the minimum, maximum or optimized area. Also it has no effect on either the minimum or maximum number of units and a weak effect on the number of optimized units. The optimized total cost increases with the increase of $\alpha$-cut.

\begin{table}[h]
\centering
\caption{The results of the regression equation}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
\textbf{Function} & \textbf{Figure} & \multicolumn{2}{c|}{\textbf{Minimum}} & \multicolumn{2}{c|}{\textbf{Maximum}} & \multicolumn{2}{c|}{\textbf{Optimized}} \\
\cline{3-8}
 & & \textbf{Slope} & \textbf{Intercept} & \textbf{R$^2$} & \textbf{Slope} & \textbf{Intercept} & \textbf{R$^2$} \\
\hline
Area & Ex1 Fig. 4 & 4.035 & 78.01 & 0.952 & -86.95 & 677.5 & 0.999 \\
 & Ex2 Fig. 10 & -2.362 & 161.1 & 0.136 & -30.51 & 652.1 & 0.790 \\
 & Ex1 Fig. 5 & - & 4 & \infty & - & 12 & \infty \\
 & Ex2 Fig. 11 & - & 5 & \infty & - & 23 & \infty \\
Utility & Ex1 Fig. 6 & 186.8 & 700.2 & 0.844 & -250 & 2460 & 1 \\
 & Ex2 Fig. 12 & 576.2 & 2913 & 0.619 & -328.8 & 7765 & 0.772 \\
Total Cost & Ex1 Fig. 7 & - & - & - & - & - & - \\
 & Ex2 Fig. 13 & - & - & - & - & - & - \\
\hline
\end{tabular}
\end{table}

NOMENCLATURE

(i) Indices:
- $i$ = hot process or utility stream,
- $j$ = cold process or utility stream,
- $s$ = index for stage $1, \ldots, \text{NOS}$ and temperature location $l, \ldots, \text{NOS} + 1$
- $cu$ = cold utility
- $hu$ = hot utility
- $in$ = inlet
- $out$ = outlet

(ii) Sets:
- $HP = \{i | i$ is a hot process stream$\}$
- $CP = \{j | j$ is a cold process stream$\}$
- $HU = \text{hot utility}$
- $CU = \text{cold utility}$
- $ST = \{s | s$ is a stage in the superstructure, $s = 1, \ldots, \text{NOS}\}$

(iii) Parameters:
- $\bar{T}_{i,\text{in}}, \bar{T}_{i,\text{out}}$ = Fuzzy inlet and outlet temperature of hot stream,
- $\bar{T}_{j,\text{in}}, \bar{T}_{j,\text{out}}$ = Fuzzy inlet and outlet temperature of cold stream,
- $FCp$ = heat capacity flow rate,
- $U_{i,\text{out}}, U_{i,\text{in}}, U_{\text{hu},j}$ = overall heat transfer coefficients
- $\text{NOS}$ = Total number of stages
- $\Omega_{i,j}$ = an upper bound for heat exchange, can safely be set to the smallest heat content of the two streams involved in the match
- $\Gamma_{i,j}$ = an upper bound for temperature difference; which are calculated based on the inlet and outlet temperatures of the superstructure,
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Operations Research and Decision Support Track

\[ \Delta T_{\text{min}} \]

= exhanger minimum approach temperature difference (EMAT).

GAMS = General Algebraic Modeling System

AHP = Analytical hierarchy process

REFERENCES


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